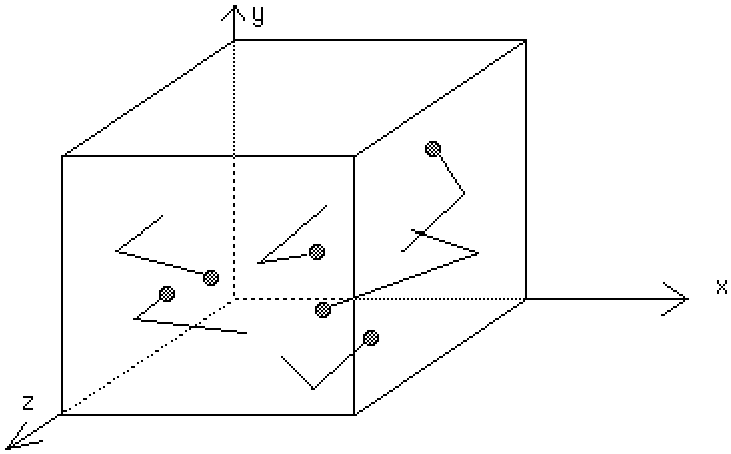
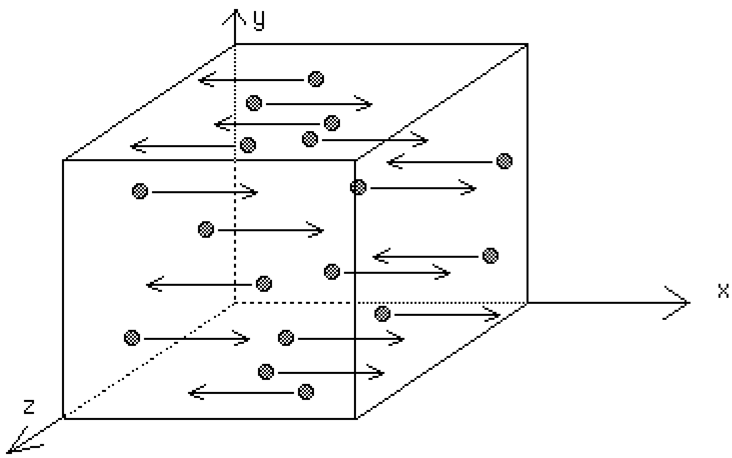
Derivation of KE = 3/2 RT



$KE = \frac{1}{2} mv^2$ (for one object or one particle)
{KE = Kinetic Energy, m = mass, v = velocity}

Velocity will be in all 3 coordinates (x, y, and z).



At this point, only the x coordinates of motion for the particles will be considered!

$p = mv$
(p = momentum, m = mass, v = velocity)

Since the particles are moving both right and left inside the box, the momentum needs to go in 2 directions (back and forth), so $p = 2mv$.

Since the particles are moving only on the x coordinate, it will be v_x.

$\mathbf{p = 2mv_x}$

In the cube, the edge will be ℓ (for length).
The particle is moving edge to edge, so the distance = ℓ.
velocity = distance over time, so $v_x = \ell / t$

Solving for time, $\mathbf{t = \ell / v_x}$

Force = $F = p / t$ (since $p = mv$, $F = p/t = mv/t$ acceleration = $a = v/t$, so $F = mv/t = ma$ and it is known that $F = ma$)

$F = p/t$ substituting in $p = 2mv_x$ and $t = \ell / v_x$

$F = (2mv_x) / (\ell / v_x) = \mathbf{2mv_x^2 / \ell}$ (combine v_x) This is the force for one molecule/particle.

For multiple molecules or particles, the equation needs to be multiplied by N for the number of molecules.

$\mathbf{F = N \cdot 2mv_x^2 / \ell}$

Since the velocity is only using x coordinates, the pressure would also only be using x coordinates.
This would equal P_x. Pressure = Force / Area

The area where the particles would hit, would be one side of the box.
Area = length X width, but since it is a cube, the area would be edge X edge, or ℓ^2.
Since there are two sides (or ends) of the box, that the particles, moving in the x coordinates, would hit, the area needs multiplied by 2. Therefore, the area of the two ends of the box = $2\ \ell^2$.

P_x = Force / Area = $(N \cdot 2mv_x^2 / \ell) / (2\,\ell^2)$ = $\mathbf{N\,m\,v_x^2 / \ell^3}$ (canceling the 2's, combining the ℓ's)

This would be the Pressure in the x direction.

$\mathbf{P_x = N\,m\,v_x^2 / \ell^3}$

But in reality, the particles are moving in the x, y and z directions. The total pressure would include particles moving in all three directions, so the pressure in the x direction now needs to be modified for all three directions.

Since P_x is the pressure from only one third of the directions of particle movement, the total pressure would be 3 times the pressure caused by particles moving only in the x direction.

$P_{total} = 3\,P_x$ (or you can think that $P_{total} = P_x + P_y + P_z$ and since P_x would equal P_y and equal P_z, then P_{total} would equal 3 times P_x)

The same applies for getting the total velocity, $v_{total} = v_x + v_y + v_z$, so v_{total} would equal 3 times v_x)

$v_{total} = 3v_x$ so $v_x = v_{total} / 3$

Substitute $P_x = N\,m\,v_x^2 / \ell^3$ into $P_{total} = 3\,P_x$ and then $v_x = v_{total} / 3$ in for v_x

$P_{total} = 3\,P_x = 3\,N\,m\,(v_{total}/3)^2 / \ell^3$ = $\mathbf{1/3\,N\,m\,v_{total}^2 / \ell^3}$ simplify the 3's: 3 X (1/3)2 = 1/3

Or for total P and v: $\mathbf{P = 1/3\,N\,m\,v^2 / \ell^3}$ multiply by 3 and

$3\,P = N\,m\,v^2 / \ell^3$ volume = length X width X height or in a cube edge^3

Volume (V) = ℓ^3, substitute V in for ℓ^3

$3\,P = N\,m\,v^2 / V$

Multiply by V

$3\,P\,V = N\,m\,v^2$

Divide both sides by ½, so the right side looks like KE

$3/2\,P\,V = ½\,N\,m\,v^2$

Substitute KE in for ½ mv^2, since KE = ½ mv^2

$3/2\,P\,V = N \cdot KE$

P V = n R T for one mole of molecules.
For one molecule: P V = N k T, where N is one molecule and the k would be a constant for one molecule.

Substitute N k T in for PV, since PV = N k T

$3/2\,N\,k\,T = N \cdot KE$

Cancel N

$3/2\,k\,T = KE$ for one molecule

Since P V = n R T for one mole of molecules, R can be used in place of k, the constant for one molecule.

$KE = 3/2\,R\,T$ for one mole